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ABSTRACT

This report is a follow-up to the report presented by the Advisory Committee on Mathematics to the California State Curriculum Commission in 1962. Recommendations are made for continued studies, experiments, and evaluations by the State Department of Education in the areas of mathematics teaching and the in-service education of teachers. Special consideration should be given to (1) the goals of an elementary school mathematics program, (2) the climate in the classroom, (3) the mathematical content of each "Strand", (4) the kindergarten, (5) proficiency in mathematical techniques, and (6) development of new evaluative instruments. (RS)

MATHEMATICS PROGRAM, K-8

1967-1968 STRANDS REPORT

PART 1

BY
Statewide Mathematics Advisory Committee

PRELIMINARY

CALIFORNIA STATE DEPARTMENT OF EDUCATION
Sacramento 1967

MATHEMATICS PROGRAM, K - 8

1967-1968 STRANDS REPORT

PART I

Report to

California State Board of Education
and
California State Curriculum Commission

by

Statewide Mathematics Advisory Committee

Preliminary

August, 1967

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Section I

INTRODUCTION

It has been five years since the first report of the Advisory Committee on Mathematics to the California State Curriculum Commission was prepared. That report gave major emphasis to a change in the content of the mathematics curriculum in grades K-8. It recommended that (1) more emphasis be placed on the structure of mathematics, (2) the elements of synthetic and coordinate geometry assume a central role in the curriculum, (3) the language of sets be introduced, and (4) mathematics and its applications should be related to the entire curriculum. We endorse that report as a sound statement of the core of the elementary mathematics curriculum. Our present recommendations will refine and revise statements concerning the cognitive aspects of the curriculum and have more to say on pedagogical requirements and the climate within the classroom.

The 1962 report was far ahead of its time in many ways. This farsightedness created many problems, two of which were very crucial. First, there was no textbook series which contained the entire mathematical content recommended in the report. Second, teachers had not had the opportunity to develop an adequate background for teaching a program in mathematics based on the recommendations. Today California can expect to secure instructional materials that will meet the standards of the Strands report. Many teachers in California have had pre-service and in-service education requisite for the implementation of the program while others are now ready and willing to undertake the additional in-service education that is necessary.

Based upon these premises, we recommend that the State Department of Education undertake a variety of experiments in the teaching of mathematics and in the in-service education of teachers. These experiments will provide school districts with suggestions for classroom and school organizational patterns which result in more effective utilization of the specialized training of teachers and with guidelines for in-service classes that will enable them to teach the mathematics program outlined in this report. We further recommend that the pre-service curriculum for elementary teachers be expanded to include a minimum of six semester or nine quarter units in mathematics.

Studies and experimentation of the last five years indicate content that can and should be taught to elementary pupils as well as effective ways in which pupils learn that content. Much of this work has been done within our own state. We view it as an endorsement of those recommendations of the first Strands report having to do with the mathematical content of the curriculum. This work and reports such as those from the Cambridge Conference on School Mathematics establish a trend toward a broader mathematics curriculum in the K-8 program. The revisions and expansions of the content of the Strands in this report are evidence of our commitment to this trend. A detailed account of these Strands is given in Section 4 of this report and constitutes our recommendation for the content of the mathematics curriculum in grades K-8 for the next five years.

We believe the most significant feature of the ongoing studies in mathematics education lies in pedagogy: How should mathematics be presented so that it will be best understood and most efficiently mastered? A growing body of evidence supports the belief that mathematics learning must involve each pupil in a participative activity employing manipulative materials as well as pure cognitive exercises.

Mathematics must be closely tied to its applications in every other discipline, from art to zoology. On the other hand, by its very nature, mathematics is abstract. We want each pupil to share this abstraction, not just for abstraction's sake, but so that he can use these abstract ideas in any situation which he may meet. The ability to generalize develops capability to solve a variety of problems in all disciplines.

There are many valid ways to teach mathematics based upon the different modes in which we learn. The final word on mathematics education will never be written because society places new demands on mathematics. Moreover, changes in methods of instruction will also have a significant impact upon the total educational process. This report is a statement of some of the things we consider important now and indicates how some of them may be achieved. We encourage continued studies, experiments, and evaluations in this field so that the resulting evidence and judgements may be used as a basis for future considerations in the development of a more effective instructional program in mathematics for boys and girls in California.

Section II

BROAD GOALS

The elementary school mathematics program shall be concerned with basic, pervasive and fundamental mathematical concepts and skills. Priority rests with those which will be used either directly or as prerequisites to other principles and techniques in mathematics, the physical and social sciences, and, as well, to the ordinary day-to-day decision making of every citizen.

Not all our pupils will be enthusiastic learners of mathematics and often they will elect not to study mathematics only to realize at a later time that this mathematics was an irreplaceable ingredient to further learning. The program must encourage, entice and cajole all pupils into doing as much mathematics as they can. It is particularly important that the mathematics program be sufficiently flexible to accept at any time new expressions of deepened interest and activity in mathematics.

The goals of the elementary school mathematics program are two fold. First, for those who will terminate their mathematical education at grade 8 or 9, the program must provide the mathematics that an informed citizen must know. Second, for those who will continue to elect additional mathematics courses in the high school, the program must provide a strong background to enable them to be successful in their advanced work. (We envision that in a few years most college capable students will complete a full year of calculus in high school.) In either case the program needs to be presented so that pupils will become excited about mathematics and develop an intellectual curiosity about and a spirit of inquiry related to mathematics. To do these things pupils need learning experiences which give them an opportunity to explore, investigate, create and recreate mathematics.

The following list of broad topics outlines what these pupils should know and *appreciate* by the end of grade 8. Naturally, different pupils will have different levels of understanding and proficiency and not all pupils can be expected to command knowledge in all these areas.

1. A sound background for algebra, including:
   Numbers and operations with numbers
   Conventional algorithms
   Sets and Functions
   Mathematical sentences
   Linear functions in a single variable
   Solution of linear equations and inequalities
   Quadratic equations, quadratic formula, and the role of the discriminant

2. A sound background for geometry, including experience with:
   Basic geometric configuration of plane and solid geometry
   Simple straightedge and compass constructions
   Congruence and similarity for plane figures
   Translations and rotations
   Measurement
   Coordinate geometry in one, two and three dimensions

3. An appreciation of the development of mathematical systems, including:
   The real number system
   Logical thinking
   Work with axioms and experience with deductive systems
   Strategies and tactics for problem solving

4. The ability to use and apply mathematics, including:
   The creation of a mathematical model as a description and means of analysis of a problem and the evaluation and interpretation of the result
   Measurement
   Statistics and Probability
   Extreme value problems; determination of maximum and minimum, both absolute and subject to constraints
   Experience with desk calculators

Section III

THE CLIMATE IN THE CLASSROOM

Perhaps the most significant feature of scholarly endeavor, be it mathematics, science or the humanities, is its spirit of free and open investigation. There should be an infusion of this spirit into the classroom for it has important implications for good pedagogy, especially for mathematics.

Mathematics and the learning of mathematics is a many faceted enterprise. While the instructional program has certain goals, each pupil and his teacher must feel free to express and explore those facets which have particular meaning to him. The most striking feature of the best presentations of mathematics is the establishment of a classroom climate under the direction of the teacher which is pupil oriented, self-directed, and non-authoritarian in concept. In this climate the teacher drops the role of an authoritative figure who passes judgement on what is right and wrong. Instead, he assumes the role of a guide who follows the leads of his pupils into uncharted regions. Following these leads, a teacher should frame questions which excite curiosity and encourage the pupils to exploit what they know and feel about the problem. The teacher can dare to follow those leads even if he does not know whether they will be productive.

There is an analogy between the teacher in the classroom and the guide on a safari. The guide has been around these wilds before and knows many of the pitfalls. He has the goal and major routes in mind. He also knows that his client wants to experience the thrill of the hunt and capture. And while the guide does not always direct his client down well-traveled expressways he does need to intervene when mortal danger is imminent. He needs to redirect his client when he has lost the way, to encourage him when weary, and provide those extra insights which will heighten the satisfaction and impact of the moment of truth. Perhaps above all, the guide must act as a pacer and he must see that his client gets to capture the big game. The teacher, too, is a guide and his pupils are the clients. Only the goal and the tools of pursuit are changed.

The classroom climate of which we speak certainly includes the spirit of "discovery" and includes a variety of ways in which pupils may direct their own learning under the guidance of an open-minded teacher. Self-directed learning requires pupil involvement in creative learning experiences which are both self-motivated and teacher-motivated. We are aware that these are not always accomplished by the "Now here's how it goes" lectures. We wish to encourage originality in the solution of problems. This requires us to realize that there is no one way to solve a given problem and that correct answers may be given in different forms. Recognition is given to learners for their own productive thinking even when it differs from that in the mind of the teacher.

In particular, the teacher should not be afraid to say, "I don't know the answer to your question". It is a healthy experience for a pupil to see a teacher struggling with a problem, especially if the teacher must make repeated attempts at a solution. A pupil can learn from listening to the teacher "thinking out loud" about the problem as well as assisting the teacher in solving the problem even though the teach-pupil team does not eventually solve the problem. From such cooperative ventures better pupil-teacher relationships are built.

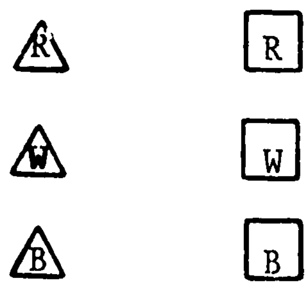
The members of one elementary class will spend approximately 1000 hours together during one school year. Thus learning is also a group experience. We would point out that this group behavior affects the learning process and that pupils do learn from one another. Mathematics becomes a vibrant vital subject when points of view are argued. We would encourage frequent cooperation between students. As pupils build mathematics together, they develop special pride in what they do and their work gains impetus. Posing a challenging problem will often serve as a special impetus for group projects as well as for individual research. These problems can come from any aspect of mathematics and sometimes serve as opening invitations for work in new areas.

One of the exciting and effective ways of facilitating learning is through the use of manipulative materials. The best of these materials are often simple things which the pupil may collect or make himself. To manipulate sometimes means pushing a pencil or drawing a figure, but more often it means handling an object, comparing objects or placing objects in various relations to each other. Pupils get a better feeling for the set concept through manipulating a collection of objects than from simply looking at a picture.

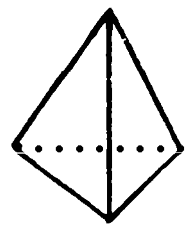
Learning is also facilitated through games, through the analysis of experiments and principles from the physical and social sciences as well as the humanities and previous mathematical experiences. The experiences should capitalize on each pupil's natural desire to see "why it works", to understand "how it works", and to see "what comes next". Here are seven examples of ways to facilitate learning in the modes we have listed above.

1. Sets, number, arithmetic operation and geometry. Each pupil might construct a set of congruent triangular cards, some which he might color red, some white and some blue and a set of congruent square cards, some colored red, some white and some blue. These could be used for example to construct a set of triangles, and then a subset of red triangles. Sets of these figures are handy for illustrating addition and subtraction, multiplication and division. We illustrate just one of these: The concept of multiplication as an array. If there are three colors of each of two shapes how many different colored shapes are there?

3 x 2 = 6



2. Geometry. Certain regular polygons (triangles, squares or pentagons) can be joined to form regular polyhedra. We use red, white, or blue triangles and join four of them to form a tetrahedron.



If there are four of each color, how many differently colored tetrahedrons can be formed?

3. Games. Consider a game in which each player is given two numbers. The winner is the one that can get the best approximation to, say, the sum of two numbers correct to within 50 (or 3, 100, or 500, depending on the level). Example: 275 + 517 is approximately 300 + 500 = 800. Such a game will provide skill in estimation and number sense, a skill which is often overlooked in the desire for excessive accuracy. We would envisage that such a game would be introduced at a time when there seemed to be some need for the pupils to know the approximate sum of two numbers.

4. Applications. A classroom experiment in growing radishes. To describe the growth we can record the data, make a graph, and utilize the mathematical concepts of function, rate of change (of growth), percent (of germination of seeds), and linear and multilinear relationships.

5. Measurement: an intuitive grasp of conversion of units. Each pupil should be able to experience the problems of measurement in each of the differing systems. A pupil can best see the need for and carry out the conversion of pounds to grams if he has handled both gram and pound weights in his hand and so compare their heft.

6. Geometry and arithmetic. Explore the relationships between the number of sides, vertices and diagonals of regular polygons. Explore the relationships between the number of edges, vertices and faces of regular polyhedra.

7. The number plane. (i) Plot a sequence of points, represented by pairs of numbers, and connect them in sequence by straight lines to form interesting looking polygons. (ii) Compute the area of a triangle as a function of the coordinates of the three vertices (for simplicity keep one edge of the triangle parallel to one of the coordinate axes). (iii) Relate multiplication by 2 and a line through the origin of slope 2.

We are aware that many of our requirements do not fit into the classical mold for textbooks. Indeed the pedagogical principles we recommend change the role of a textbook as something each child should read and understand passively, to a source book for experiences which each child undertakes actively.

We hope that we shall see among the materials submitted for adoption many ways of breaking through the traditional textbook barrier to learning. Certainly careful consideration must be given to the amount of verbalization and the reading skills required by the pupil to understand his text. It is a paradox that mathematical concepts themselves are non-verbal yet communication of these concepts requires sophisticated verbal skills and ability to use and understand symbolic representations. A delicate balance must be maintained and there should be a greater emphasis on the mathematical concepts themselves than on vocabulary or a pedantic insistence on certain symbolic conventions. On the other hand, the introduction of common mathematical terms or practices should not be overly delayed. For example, there is no reason why literal symbols cannot be employed as well as frames at an appropriate level.

Materials adopted by the State to implement the mathematics program must be sufficiently flexible to be used with a variety of teaching methods and organizational plans. The ways in which the text can be personalized to the needs of the individual must be delineated. Whether or not ability grouping takes place, it is clear that in any classroom the rates of learning will vary and the pacing of instruction must be planned accordingly. Perhaps of more significance, the pupils' modes of thinking will differ; some think best in concrete terms, others in abstract formulations. The introduction of a new mathematical concept must be done in such a way as to appeal to each of these ways of thinking.

Section IV

AN OVERVIEW OF THE STRANDS

In this section, we present a brief indication of the mathematical content of each Strand. Those familiar with the first Strands report will have little difficulty in noting the revisions and extensions we suggest. Fuller explanations of the technical terms we use as well as our ideas on the sequence and depth of the mathematical concepts, the appropriate vocabulary, and the symbolism are detailed later.

Strand 1. Numbers and Operations

The content of this strand is the heart of a traditional program of mathematics and it remains central in the learning of pupils in Grades K-8.

We recommend that development of the various number systems show their structure as a sequence of expanding systems from the counting numbers through the rational numbers. The properties of the order relation should be studied on the number line. Such study relates to strands of geometry and numbers, a relationship that must be strengthened by use of the number plane.

The four fundamental binary operations of addition, subtraction, multiplication, and division must be presented. We recommend that subtraction be treated as the operational inverse of addition, and division as the operational inverse of multiplication. Other interpretations of subtraction and division are also important for mathematical algorithms and applications. Attention should be given to the properties of commutativity, associativity and distributivity and to the identity elements 0 and 1.

We recommend that the system of rational numbers be sequentially developed to show that:

1. The rational numbers are constructed from the system of integers.

2. A rational number is denoted by a fraction and that many fractions denote the same rational number.

3. The operations of addition and multiplication are defined so that the properties which hold for the integers remain true.

4. The subtraction of two rational numbers is viewed as a solution for an equation of the form $a + x = b$; similarly, division must be viewed as a solution for an equation of the form $ax = b$.

5. Properties of order, betweenness, and density are given geometric interpretations.

6. Not all points on the number line can be denoted by rational numbers.

We recommend that preparatory experiences for study of the full set of real numbers include associating a real number with each point on a line and using decimal notation as a way of denoting a real number.

We recommend that the study of numeration systems be illustrated with a brief treatment of bases other than ten. This treatment should show the role of the place value numeration system and the invariance of the properties of the numbers under a change of notation.

Strand 2. Geometry

The K-8 mathematics program should provide a strong intuitive grasp of basic geometric concepts: point, line, angle, plane, three dimensional space, congruence and similarity, and coordinate geometry. The development of geometry must progress at each grade level. We do not envisage a sequence of definitions, theorems and proofs but rather a wide assortment of informal geometric experiences. There must be a continual tie-in with the strands of number, measurement, applications, problem solving, and logical thinking. There will be times when short chains of deductive reasoning will be both time saving and instructive for pupils.

Strand 3. Measurement

Measurement is a "doing" process and is best learned in this context. It provides a way to use the mathematical concepts and skills found in the other strands as well as a source of information for teaching certain skills and concepts. Measurement is first presented as a way of comparing a common attribute (distance, area, number, probability) of two things. Second, arbitrary units are selected for measuring. Finally, standard units are used as a means of communicating ideas so that there can be universal understanding.

We recommend that measurement be presented within the central point of view which recognizes that:

1. Measurement is a function which assigns a number to an object to reflect a property of the object.

2. A unit is selected which has the same property as the object to be measured. Then a comparison is made and the number of these units present in the object is determined.

3. The choice of the unit is arbitrary.

We recommend that the standard mensuration formulas be developed from the three points mentioned above.

We recommend that the English and metric systems be introduced in such a way that pupils will become bilingual in their use and learn the advantages and disadvantages of each system. Pupils should develop a comparative sense about the two systems, e.g., a centimeter is less than, but about one-half inch.

Strand 4. Applications of Mathematics

We recommend that concrete problems from the physical and social sciences be presented hand in hand with pure mathematics. Constant touch with honest and concrete situations serves two purposes. First, it provides an opportunity to use the mathematics and the skills already developed to attack interesting problems. Second, it serves as a motivation for new and deeper mathematical insights. We recommend that application of mathematics be presented as a process which

1. Constructs a mathematical model which reflects significant properties of the concrete problem.

2. Formulates and analyzes a mathematical problem.

3. Interprets the results of the mathematical analysis.

We particularly urge that the adopted materials suggest ways for teachers to help pupils propose areas for investigation which are of interest to them.

Strand 5. Statistics and Probability

We live in a society in which data abounds. Mathematics provides techniques to synthesize a conglomeration of numbers and extract a few quantities which indicate the overall situation. Often this process can be used effectively to predict behavior in similar and analogous situations. The scope, power, and limitation of statistics and its theoretical sister, probability, constitute vital knowledge for every citizen.

In the elementary grades we recommend that our pupils learn:

1. The rudiments of organization of data into standard graphs and charts.

2. The meaning of average (mean), median, and mode.

3. The meaning of variance and standard deviation as a crude measure of how much of the total distribution is "close" to the average.

4. The elementary notions of probability as it pertains to "laws of chance" and to the physical situations from nature for which these are the correct mathematical models.

5. The elementary notions of statistical inference.

Strand 6. Sets

We recommend an early introduction of the set concept to express the elementary notions of one-to-one correspondence and number. In this way a universal rationale can be given for the important properties of the integers. It is imperative to continue to use the language of sets in studying geometry, functions, solutions of equations and inequalities, number theory, and graphing. The set operations required are union, intersection, complementation, and Cartesian product. The properties of the set operations of union and intersection required are commutativity, associativity, and distributivity.

We recommend that the language of sets not be invoked for itself, but rather that it be used to clarify and make precise the concepts and applications of mathematics. In particular, we warn against the pedantic practice of requiring set terminology when a simple verbal phrase would suffice.

Strand 7. Functions and Graphs

The function concept permeates all of mathematics as well as the real world. In intuitive terms, whenever one quantity is determined by others, a function is involved. For example, the area of a square is a function of the length of its side; the volume of a cone is a function of the radius of its base and its height; the speed of an automobile is a function of its acceleration, which in turn may be a function of the time the car is running. Functions are often pictured by graphs, in one, two, or several dimensions. Functions and their graphs form powerful tools for studying mathematics and its applications.

We recommend that the function concept, and its generalization, the relation concept, be developed, named, and used in the elementary school program. Graphs of functions and relations in one and two dimensions should be introduced in the early grades. In grades seven and eight, the standard functional notation, which denotes the value of the function, F, at the object, x, as f(x), or prescribes the function, f, as

$$f : x \to f(x)$$

should be employed.

Strand 8. Logical Thinking

There are two sides to a mathematical investigation; the inductive and deductive. In Grades K-8 we emphasize and exploit the inductive sides; we build experience with mathematical concepts. But the strength of mathematics comes from its deductive side. From certain assumptions we deduce and infer other properties and behaviors. This, too, is a mathematical experience we want each pupil to share and appreciate. We can best summarize our attitude toward logic and deductive reasoning as "horse sense". We want our children to understand:

1. The logical connectives, "and" and "or".

2. The meaning of the "If A, then B" sentence and the rule of inference which yields "B" if <u>both</u>
   "If A then B" and "A" have been established.

3. The role of negation.

4. The role and scope of the quantifiers
   "For all ..." and "There exists a ..."

5. The notion of a "proof" as distinct from a "check".

6. That equality in mathematics always means two different names for the same object or number.

This <u>Strand</u> indicates how this can be accomplished without a heavy, formal logical system.

<u>Strand 9. Problem Solving</u>

This <u>Strand</u> recognizes that one of the major objectives of the mathematics program is the formulation and solution of problems. There is a sharp distinction between the process of applying mathematics through the construction of mathematical models and the techniques of analysis of mathematical problems. A pupil learns much of his mathematics while he is engaged in the latter activity; he learns its significance while engaged in the former.

We recommend that pupils learn to use different strategies and tactics for general problem solving such as:

... The construction of a diagram or the use of materials to illustrate the problem.

... Guessing a reasonable answer.

... The translation of the conditions of the problem into mathematical sentences.

... The performance of the mathematical analysis and the interpretation of the answer.

Any list of strategies or sequential steps in problem solving should be viewed as _optional_. Under no circumstances should a pupil be forced to do a problem in a fixed way if he has discovered a method of his own which brings the problem to a correct solution.

Section V

THE KINDERGARTEN PROGRAM

The powerful, interrelated ideas of mathematics have their genesis in the experiences of young children. These early experiences provide intuitive background essential to the development of later mathematical concepts. Therefore, we recommend that the instructional program in mathematics begin in kindergarten.

The kindergarten program which we envision is informal and exploratory in nature. For such a program to be effective, the teacher must have clearly in mind both its objectives and many possible ways of achieving them. We recommend that each kindergarten teacher be provided a guide discussing ways in which young children begin to develop mathematical ideas. This guide should give teachers specific assistance in planning suitable learning experiences. Because of different organizational plans in the schools and of different needs in various regions of California, the guide for kindergarten should be made available also to districts that request it for teachers of the first grade.

The heart of the program should be activities involving children with physical objects that are usually found in the kindergarten environment. For example, these activities might use blocks, objects in the playhouse or store, toys, pegboards, kindergarten beads, paints and brushes, clay, balls and playground equipment, things brought for sharing or collected on neighborhood walks, rocks and shells, plants and animals or story books. Other activities may be centered around materials especially designed to develop certain mathematical ideas. The experiences should give children opportunities to compare objects; classify and arrange them according to such attributes as shape and size; experiment with symmetry and balance; and discover, continue, and create patterns. Children should discover the relations of more, fewer, and as many as, through the activity of matching <u>small</u> sets of objects. These experiences promote the development of such concepts as one-to-one correspondence, number, and the ability to count. They should be able to name the cardinal number of a set, count at least through ten, and develop positional relationships such as inside, outside, on; first, next, last; before, after, between; left, right; above, below.

Throughout their activities children should be encouraged to ask questions and talk about what they are doing both with the teacher and among themselves. Children at this level are imitative, interested in words, and rapidly increasing their vocabulary. If the teacher introduces word and language patterns easily and naturally in situations that make their meaning clear, children will begin to assimilate the word and patterns into their own speech and thoughts. The key words here are <u>easily</u> and <u>naturally</u>. Children's own ways of conveying their ideas must be accepted while at the same time opportunities must be provided for learning to express them with greater clarity and precision.

Section VI

TECHNICAL PROFICIENCY

Facility and precision in arithmetic as well as algebraic and geometric techniques are requisite for further mathematics and scientific work. Certainly every pupil will have to be able to perform the four elementary arithmetic operations, to solve simple equations and inequalities, to make elementary geometric constructions, and to draw graphs.

It is important to note that the types of mathematical techniques required by every adult and the frequency with which he uses them are rapidly changing. Business and industrial procedures involving even the most rudimentary arithmetical operation are now either done or checked with a calculator or computer. It is less clear how many, and to what degree of complexity, arithmetic operations and the skillful performance of the associated algorithms will permeate the daily life of tomorrow's citizen.

For these reasons those skills we select for precision and speed drills must tie into the whole mathematics curriculum. To implement the pedagogical philosophy we have delineated, these drills must not be permitted to degenerate into stultifying and time consuming routines which stifle instead of nurture the receptive minds of elementary pupils. It is, of course, true that we require a degree of mastery of the facts and algorithms which enable pupils to think through situations without being cluttered mentally by errors.

We urge the acceptance of correct algorithms invented by the pupil, counting on the fingers or the use of addition and multiplication tables even though they may be inefficient and time consuming. Certainly we hope that pupils will eventually learn to replace these inefficient computational methods by more powerful ones. The pupil will develop better procedures if he first realizes that his methods are not wrong. Once he knows that what he is doing is right and acceptable he may be directed, or in many instances he will be self-motivated, to learn faster and more efficient methods.

We know that complete mastery of number facts and operational skills will come only after continued _use_ of numbers. This process of gaining technical skill is an ongoing process. It is our belief that the best way to acquire these skills is to impart them along with more mathematics, in connection with applications of mathematics, or with games. It goes almost without saying that a game, in place of flash cards, makes any arithmetic problem more attractive. Any of the seven examples of ways to facilitate learning cited in Section I will provide opportunities for practice in computational skills.

Attention to individual differences is never more important than in teaching addition facts. Some children learn best from tables, some by reasoning, some by regrouping for combination adding to ten. An illustration of the latter is shown here.

$$6 + 7 = 6 + (4 + 3) = (6 + 4) + 3 = 10 + 3 = 13$$

Some pupils will learn by regrouping for doubling:

$$6 + 7 = 6 + (6 + 1) = (6 + 6) + 1 = 12 + 1 = 13$$

We point out that our numeration system means that ultimately only the addition and multiplication table from 0 - 9 need be learned. As an aid in computation, we exploit commutativity and realize that only half of the facts need be learned. We are aware that understanding commutativity comes from analyzing pairs of such addition facts such as, $5 + 3 = 8$ and $3 + 5 = 8$. Therefore, it should not be necessary to drill on both of these facts since a general principle has been identified as an aid for computation.

Effective practice requires problem by problem reinforcement. Errors must be corrected as they occur. It is pointless to assign twenty exercises only to discover that one fundamental error has been repeated in half of them. Some exercises should explicitly call for practice with manipulative materials and simple geometric constructions. Cooperative work with a partner or in a small team provides another means of immediate feedback, and the techniques of programmed learning provide yet another. An acceptable textbook series will provide practice opportunities which can be adjusted to the needs of the learner. It must also provide immediate checks and reinforcements, and in this way aid the pupil in becoming responsible for his own learning.

Section VII

EVALUATIVE PROCEDURES

Any educational program requires some evaluation procedure. We emphasize here that new programs require new evaluative instruments. This is particularly true in mathematics. Traditional training in mathematics emphasized technical skills; especially arithmetic skills. These skills are relatively easy to evaluate since an arithmetic skill is a very narrow concept; a column of figures is either added correctly or not. But the new programs in mathematics, including our present curriculum, have goals which are broader in scope and seek understanding of more subtle mathematical concepts. Understanding of these concepts is not measured alone by how well a pupil performs a technique. For this reason new tests and new test criteria must be developed to measure the success of a mathematics program designed to attain a wide spectrum of ends. The design of these instruments is not an easy problem and the statistical evaluations for the reliability and validity of a particular instrument are complicated. We urge that evaluations of the new programs in mathematics rest on new evaluative instruments and we urge that as soon as possible, a special task force be created to deal with these problems.

The goals we seek are not achieved overnight or even in a single year. It may well be that performance at the secondary level after the pupils have been in the program for six or eight years may be a better index of the elementary program than a battery of tests. Unfortunately, such information will not be available for some years.

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